

## MATHEMATICAL MODELING OF SOLAR-HYDROGEN SYSTEM FOR RESIDENTIAL APPLICATIONS

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### ABSTRACT

A possible future approach to home and personal vehicle energy needs is to combine solar power generation on the structure with hydrogen production and use. In this paper, performance of a solar-hydrogen system for residential application is simulated. Included in the main components of the solar-hydrogen system are photovoltaic panels, electrolyzer, fuel cell, compressor, and storage tank. The solar-hydrogen system is assumed to be installed on the standard house (1600 sq ft), to be a part of a grid-connected system where the house can receive energy from both grid as well as PV panels and fuel cell based on the energy requirements of the house. Using yearly weather data for Las Vegas, net grid interaction as well as stand alone (off grid) performance is considered as part of the analysis. The effect of component sizing aspects is examined.

### INTRODUCTION

Many analysts have concluded that hydrogen is one of the best options as a future widespread energy carrier. Hydrogen can be used for a variety of applications including combustion-based approaches as well as direct conversion to electricity through fuel cells. Because of the temporal variation of many renewable energy sources, intermediate hydrogen may be a good way of buffering between these variations and time varying energy use patterns.

This paper describes the performance simulation of solar-hydrogen system for residential applications. The main components of the solar-hydrogen system are photovoltaic panels, electrolyzer, fuel cell, compressor, and hydrogen storage tank. The solar-hydrogen system installed on the standard house (1600 sq ft) is assumed to be a part of a grid-

connected configuration where the house can receive energy from both grid as well as PV panels and fuel cell based on the energy requirements of the house.

A grid-connected system is considered for three reasons. One is that for the residential purposes we may need a continuous supply of power that might not conveniently be available from the house-based system alone, and grid connection can satisfy that. A second reason for this assumption in the analysis is that it can allow us also to examine the zero grid draw cases, which will be similar to stand-alone configurations. Another possibility is that we could have a group of residential houses interconnected to each other and/or to the grid. The houses can then trade energy either in the form of electrical power or in the form of hydrogen as fuel.

In what follows we consider a numerical formulation of a variety of components in a solar electric/hydrogen system. The modeling is such that the sizes of the components can be varied to study their impact on the overall system performance. While a grid-connected system is considered, zero net grid draw cases are examined that are relevant to stand-alone systems. The case where the system satisfies the needs of the house is presented in this paper.

It is reported in the literature that some experimental studies were performed as well as mathematical modeling of a renewable hydrogen system for stand alone applications driven by PV only and also by a combination of PV and wind. The experimental results for a one-year period with solar hydrogen system that provides uninterrupted  $200W_e$  power to an isolated application are described [1]. Simulations were performed for the energy requirement of a solar hydrogen system of the low energy stand-alone application. The sizes and configurations presented were for dwellings located in Trondheim, Norway

[2]. The design and construction of a regenerative electrolyzer/fuel system have been reported [3]. A dynamic model was developed and comparisons were made to experimental results for a stand-alone renewable energy system (PV and wind turbines) with hydrogen storage [4]. Another paper presented a review on the domestic production of hydrogen using renewable energy [5]. A PV-electrolyzer system was studied in detail in this work. A hydrogen based integrated renewable energy system (wind turbine and PV) is presented for powering remote communication stations [6]. Reported studies for a grid connected solar hydrogen system are rare though some of the papers noted above did mention a grid-connected system.

There is need for modeling a grid-connected solar hydrogen system for residential application as it can help to study the nature of behavior of this system when used for this application. Also, the grid-connected system can be analyzed for zero energy draw from the grid, which will result in a stand-alone configuration. Hence this paper describes the modeling and sizing the components of grid-connected solar hydrogen system for residential applications.

## NOMENCLATURE

Upper case

$A$ : active area of electrodes ( $m^2$ )

$C^*$ : Concentration ( $mol\ cm^{-3}$ )

$F$ : faraday's constant

$\Delta G$ : change in Gibbs energy ( $kJ/mol$ )

$I$ : current (A)

$I_L$ : light current (A)

$I_O$ : diode reverse saturation current (A)

$P$ : pressure (atm)

$R$ : cell resistance (Ohm)/ gas constant

$R_S$ : series resistance (ohm)

$R_{SH}$ : shunt resistance (ohm)

$\Delta S$ : change in entropy ( $kJ/K.mol$ )

$T$ : temperature (K)

$V$ : voltage (V)

$V_{EL}$ : electrolyzer voltage (V)

$V_{FC}$ : fuelcell voltage (V)

$V^O$ : standard voltage, 1.229 Volts

$V_R$ : reversible voltage (V)

$W$ : power (Watts)

Lower case

$a$ : a ideality factor parameter

$i$ : current density ( $A/ m^2$ )

$i_L$ : limiting density current ( $A/ m^2$ )

$n$ : number of moles in the reaction/ Polytropic exponent

$r$ : parameter related to ohmic resistance of electrolyter ( $ohm.m^2$ )

$s$ : coefficient for overvoltage on both electrodes (V)

$t$ : coefficient for overvoltage on both electrodes ( $m^2/A$ )

Greek

$\xi_1, \xi_2, \xi_3, \xi_4$ : overvoltage parameters

$\eta$ : polarization overvoltage (V)/ efficiency

Subscript

Upper case

COMP: Compressor

EL: electrolyzer

FC: fuel cell

H<sub>2</sub>: Hydrogen gas

H<sub>2</sub>O: Water

O<sub>2</sub>: Hydrogen gas

Lower case

a: anode

act: activation

c: cathode

conc: concentration

g: gas

ohm: ohmic

ref: Reference

## SYSTEM DESCRIPTION

A house of 1600 sq ft located in Las Vegas, NV is selected for the purpose of analyzing its energy behavior. The energy requirements of the house are met by a grid-connected PV – hydrogen system. The major components of the PV-hydrogen system are a PV array, electrolyzer, compressor and storage tank, and fuel cell. Detailed information about each component is given below. Figs. 1 and 2 illustrate the system considered.

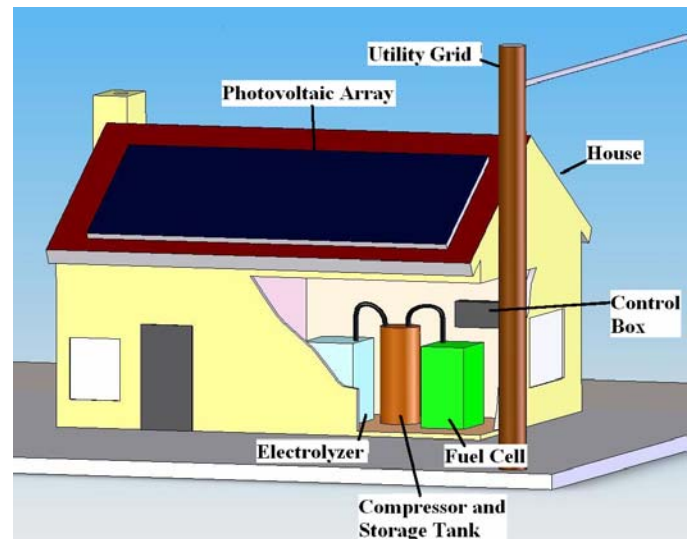


Fig. 1: Residential house with PV-hydrogen system.

### a. Weather Data

TMY2 hourly averaged weather data for Las Vegas is used for the calculations related to the atmospheric conditions. The solar irradiation and temperature data is used for modeling the performance of the PV array [7]. The Perez et al. model [8] was used to simulate the solar radiation for Las Vegas, NV.

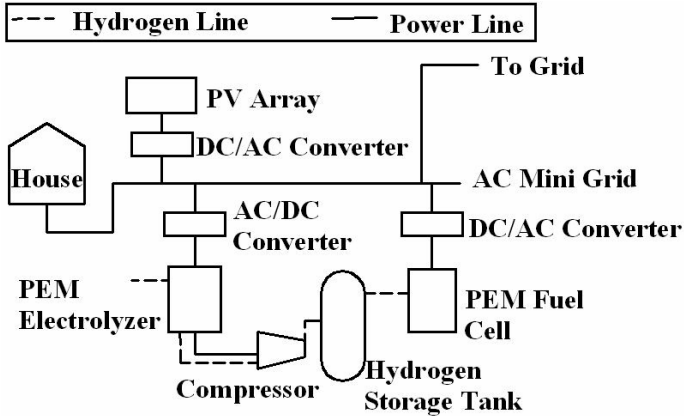


Fig. 2: Solar hydrogen system for the house.

The optimum array angle to achieve the maximum annual power generated by the PV for Las Vegas 36 degrees [15], and this value was used throughout. Fig. 3 shows the solar irradiation on a surface tilted at 36 degrees for Las Vegas.

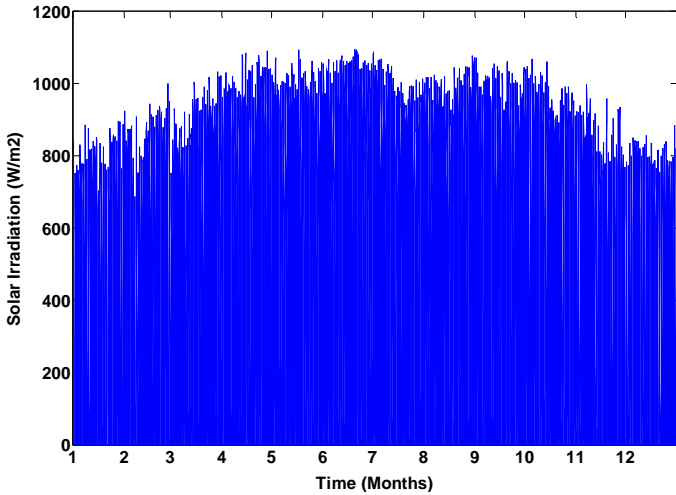


Fig. 3: Solar irradiation on surface at optimum tilted angle for Las Vegas, NV (36 deg) for one year.

The ambient temperature is also used for calculations of hourly electrical load for the house. Fig. 4 shows the ambient temperature for Las Vegas for one year.

### b. House

The house under consideration is selected such that the total energy demand is reduced by conservation features to 740 W-hr/day. The structure of the house is assumed to be of wall type having a resistance value of R-36. The roof of the home has an R-value of 50. The R-value of the floor is assumed to be 23.5. The windows have an overall heat transfer coefficient ( $U$  value) of 0.27 Btu/hr-sq ft-F.

Considering this structure for the house the overall heat transfer for the house was calculated based on the floor area as 0.0481 Btu/hr-sq ft-F. The overall heat transfer calculated takes into account the energy demand for the air conditioning

system as well as the other residential energy requirements e.g. lights, appliances, etc. Fig. 5 shows the electrical load of house for the period of one year calculated from the  $U$ -value.

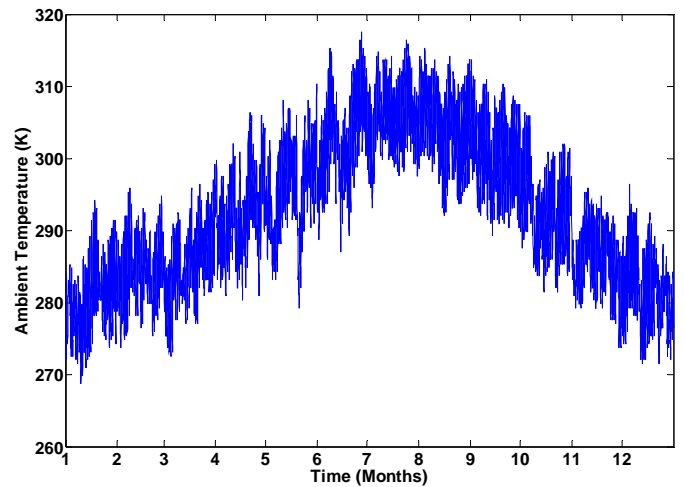


Fig. 4: Ambient temperature for Las Vegas, NV for one year.

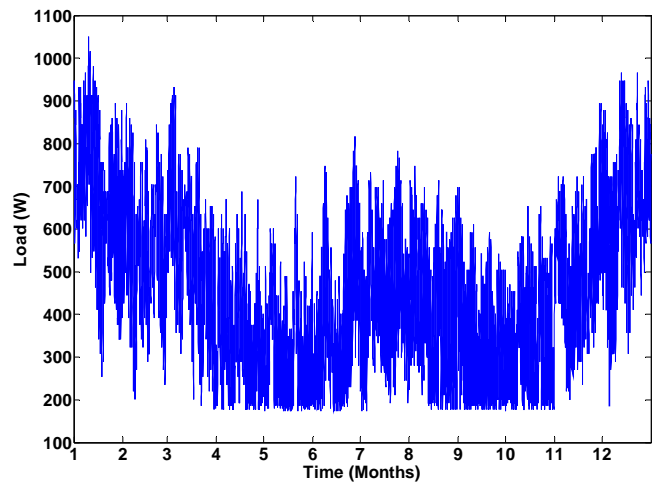


Fig. 5: Annual electrical load for the house.

### c. PV System

The mathematical model of a PV system is based on a one diode model that can be used for system design of an individual cell, for a module consisting of several cells, or for an array consisting of several modules [9]. This model needs five parameters (light current, the diode reverse saturation current, series resistance, shunt resistance and modified ideality factor) to determine the current, voltage and electrical power supplied to the load. The current-voltage ( $I$ - $V$  characteristic curve) relationship of the cells takes into account the effect of variation in radiation and cell temperatures. At fixed temperature and solar radiation, the current-voltage characteristic of a PV module can be modeled using Eqn. (1).

$$I = I_L - I_0 \left[ e^{V + IR_S / a} - 1 \right] - \frac{V + IR_S}{R_{SH}} \quad (1)$$

#### d. Electrolyzer

The Electrolyzer splits water into hydrogen and oxygen. The electrode kinetics of an electrolyzer are modeled using semi-empirical current-voltage relationships of electrolyzer ( $I$ - $V$  curve) [10]-[11]

$$V_{EL} = V_R + \frac{r}{A} I_{EL} + s \log \left( \frac{t}{A} I_{EL} + 1 \right) \quad (2)$$

where the reversible potential ( $V_R$ ) for the water splitting process is expressed using the Nernst relation [12]

$$V_R = V^0 - \frac{RT}{nF} \ln \left[ \frac{P_{H_2} P_{O_2}^{1/2}}{P_{H_2O}} \right] \quad (3)$$

For simplicity, the temperature of the electrolyzer is assumed to be constant while in operation.

#### e. Fuel Cell

A fuel cell is an electrochemical device that converts hydrogen and oxygen to electrical current (DC). The total cell potential can be evaluated using the relation:

$$V_{FC} = V_R + \eta_{act} + \eta_{conc} + \eta_{ohm} \quad (4)$$

The overall cell potential consists of reversible voltage  $V_R$ , activation overvoltage  $\eta_{act}$ , ohmic polarization  $\eta_{ohm}$  and concentration losses  $\eta_{conc}$ . Reversible voltage,  $V_R$ , of a fuel cell is obtained by an open circuit thermodynamic balance. It is calculated using the modified version of Nernst's equation, with a term that takes into account changes in the temperature in relation to the standard reference temperature, and partial pressure of the gases at 25°C.

$$V_R = \frac{1}{nF} \left[ \Delta G + \Delta S (T - T_{ref}) + RT \ln \left[ P_{H_2} P_{O_2}^{1/2} \right] \right] \quad (5)$$

The activation overvoltage is directly related to the rates of electrochemical reactions. In the case of an electrochemical reaction with  $\eta_{act} > 50$ -100 mV the total activation overvoltage (cathode and anode) can be represented by the following expression [13], [14]

$$\eta_{act} = \xi_1 + \xi_2 T + \xi_3 T \left[ \ln \left( C_{O_2}^* \right) \right] + \xi_4 T \left[ \ln i \right] \quad (6)$$

The resistance to electron transfer and resistance to proton transfer on solid polymer electrodes electrolyte membrane results in an ohmic loss that is given by:

$$\eta_{ohm} = -i \left( R^{electronic} + R^{proton} \right) = -i R^{internal} \quad (7)$$

There is a loss of voltage due to inability of the surrounding material to maintain the initial concentration of the bulk fluid due to consumption of reactant at the electrode from electrochemical reaction. Such losses are called *concentration losses* and are evaluated by:

$$\eta_{con} = \frac{RT}{nF} \ln \left[ 1 - \frac{i}{i_L} \right] \quad (8)$$

#### f. Compressor, Storage Tank, and Power Grid

The compressor is modeled assuming ideal gas undergoing a polytropic process. The work done for a polytropic process is given by [10]:

$$W_{COMP} = \frac{n_g}{\eta_{COMP}} \left[ \frac{n}{n-1} RT \left[ 1 - \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right] \right] \quad (9)$$

The storage tank model uses the ideal gas law to calculate the pressure level and amount of gas inside the tank.

$$P = \frac{nRT}{V} \quad (10)$$

A simple energy balance model is used to evaluate the amount of energy either received from the grid or supplied to the grid.

$$W_{GRID} = W_{PV} + W_{FC} - (W_{EL} + W_{COMP} + W_{LOAD}) \quad (11)$$

## RESULTS

This section describes the result of the simulations for the system discussed above. Performance calculations were conducted for a period of one year for the complete system. It can be argued that the simplest possible system that can help reduce the amount of power draw from the grid would be a house connected with only PV arrays. The PV array is not able to supply the energy demand by itself in case of bad weather or times having insufficient solar irradiation. In such conditions there should be some other means to satisfy the energy demand of the house. Hence the house with PV as well as a hydrogen system (electrolyzer and fuel cell) is considered that would be able to supply not only the power demand of the house but also the fuel demand for vehicles in near future.

The bulk of the results shown here are given for a system that does not generate excess hydrogen (denoted in what follows as a *contained* system). That is, the goal of these calculations is to balance exactly the energy generated by the PV with energy requirements of the house.

Sizing of the various components for the bulk of the calculations (no excess hydrogen sought) is described as follows:

- Selection of the fuel cell is on the basis of maximum energy requirement of the house during any time of the year. From the load calculation it was found to be 1000 W rated capacity.
- For a given fuel cell, the tank size is determined based upon how many days the fuel cell needs to be the sole source of power for the house. Two tank sizes are chosen initially considering their ability to serve the given load by itself for 16 days and 32 days respectively. The tank sizes obtained were 0.95 m<sup>3</sup> (31.8 kg) and 1.9 m<sup>3</sup> (63.6 kg) respectively at 413-bar pressure. Later we show the effect of tank size for the optimum sizes of PV and electrolyzer. For all the simulations, the initial tank level is considered as 0.5 (fraction filled) and electrolyzer and fuel cell are allowed to work between maximum hydrogen level as 0.95 and minimum hydrogen level as 0.5.
- The PV and electrolyzer sizes are determined based on the amount of energy sent back to grid (ideally this latter amount would be zero).
- We then “float” the size of the electrolyzer so as to obtain the optimum performance of the system.

Figs. 6, 7 and 8 show the performance of the different components of the system for the tank size of 0.95 m<sup>3</sup>. Fig. 6 shows the operational time for the electrolyzer for a *contained* system as a function of the PV system and electrolyzer size. It can be seen that the larger values of electrolyzers reach a peak in operational time and then decrease. However, the smallest sized unit is too small, and requires an ever-increasing amount of operation with increased PV size.

Fig. 7 shows the grid power required for a variety of system parametric variations. A null value indicates that the system generates the same amount of energy as the house requires. Approximate 3.2-3.3 kW of PV will drive all of the sizes of electrolyzers sufficient to give the null value of grid power.

Fig. 8 shows the variation in hydrogen produced for the *contained* system. Note that over the initial portion of the range examined, the electrolyzer size has small effect, but the PV power rating influences the results greatly. At large PV array sizes, the system begins to saturate, with smaller and smaller amounts of additional hydrogen produced with increasing PV size for a given electrolyzer.

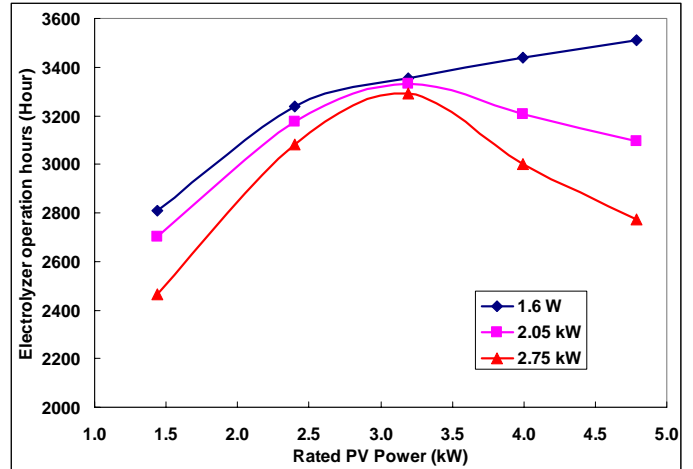


Fig. 6: The electrolyzer operational time over a year’s period for a variety of PV array sizes and electrolyzer sizes in a *contained* system.

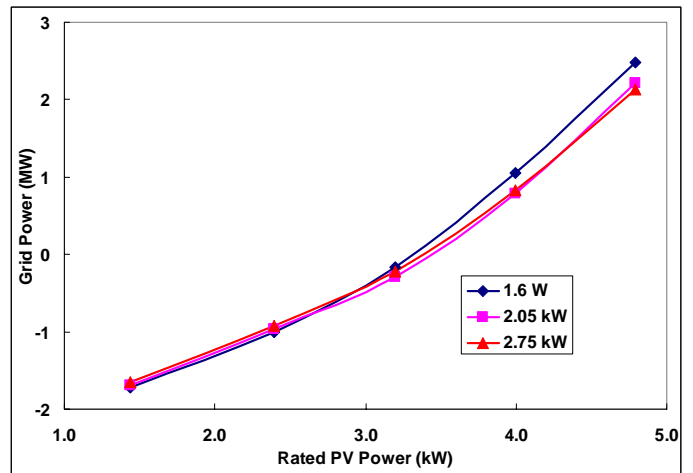


Fig 7: Grid power required for a *contained* system over a year’s period as a function of the PV and electrolyzer sizes.

Next we examine the effect of increase in the tank size from 0.95 m<sup>3</sup> (31.8 kg) to 1.9 m<sup>3</sup> (63.6 kg). Figs. 9 and 10 show the comparison of system performance between the two tank sizes. From Fig. 9 we can observe that as the tank size is doubled there is increase in the grid power for the same size of PV, but this trend is visible only after the minimum PV size of approximately 3.2-3.5 kW. Notice that by increasing the tank size there is no considerable difference between performance of 2.05 kW and 2.75 kW electrolyzers for the given range of PV sizes.

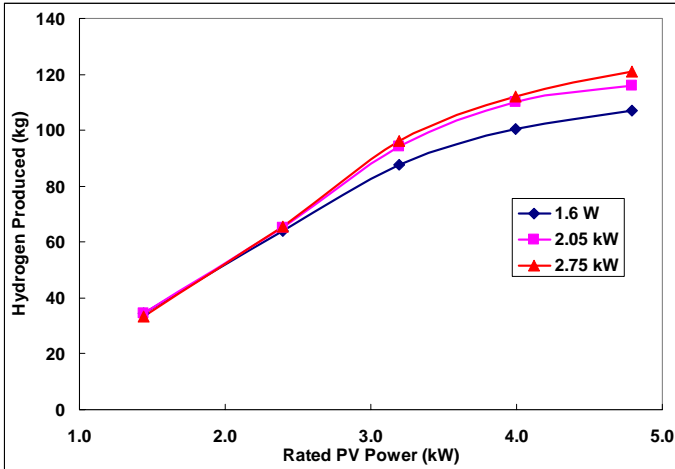


Fig 8: Hydrogen produced for a year's period for a contained system and a variety of electrolyzer and PV system sizes but with storage tank size held constant at  $0.95 \text{ m}^3$  (31.8 kg)

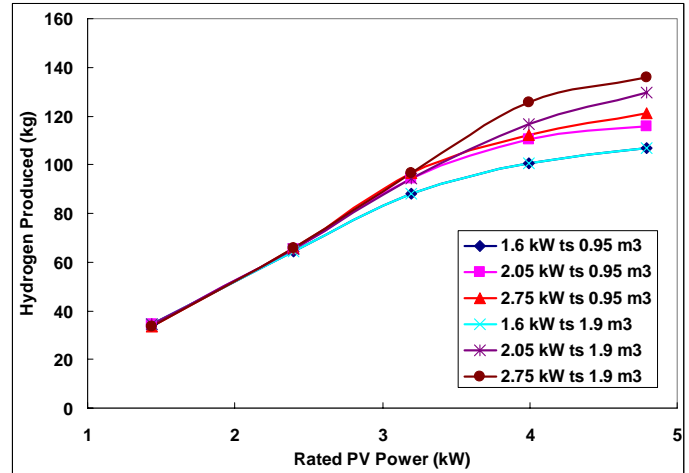


Fig. 10: Comparison of grid power required for a contained system for two tank sizes  $0.95 \text{ m}^3$  (31.8 kg) and  $1.9 \text{ m}^3$  (63.6 kg).

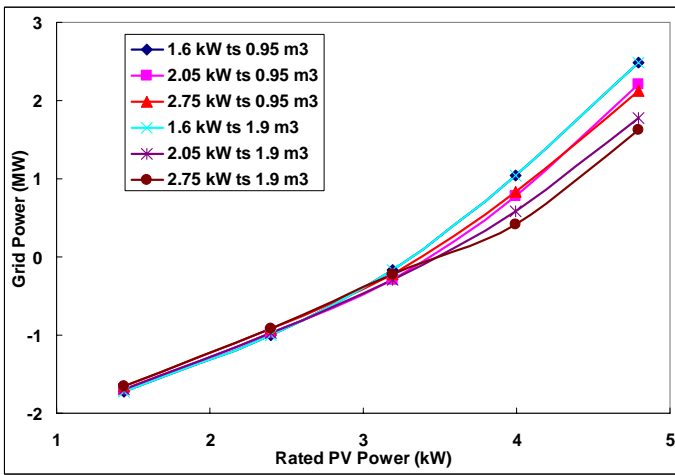


Fig. 9: Comparison of grid power required for a contained system for two tank sizes  $0.95 \text{ m}^3$  (31.8 kg) and  $1.9 \text{ m}^3$  (63.6 kg).

From Fig. 10 it can be observed that for the up to a PV size of 3.5 kW there is no significant difference in hydrogen produced for different tank size and electrolyzer sizes of 2.05 kW and 2.75 kW. Hence from the above behavior of the system we can see that the optimum size for the solar hydrogen system would be when we have PV of 3.5 kW, electrolyzer of 2.05 kW rating and the 1 kW fuel cell for the house.

Since the results discussed before show that there is change in system performance with a variation in tank size, we will further examine the effect of tank size if we hold the PV and electrolyzer size constant to reach the optimum tank size. From Fig. 11 it can be observed that as we increase the tank size the ratio of energy generated by PV to the energy consumed by the electrolyzer will drop up to the point and then remain constant. This happens because when the tank is small

then the maximum operating pressure is reached fast as compared to when the tank is larger. It is also observed that the minimum grid power is obtained at the same tank size.

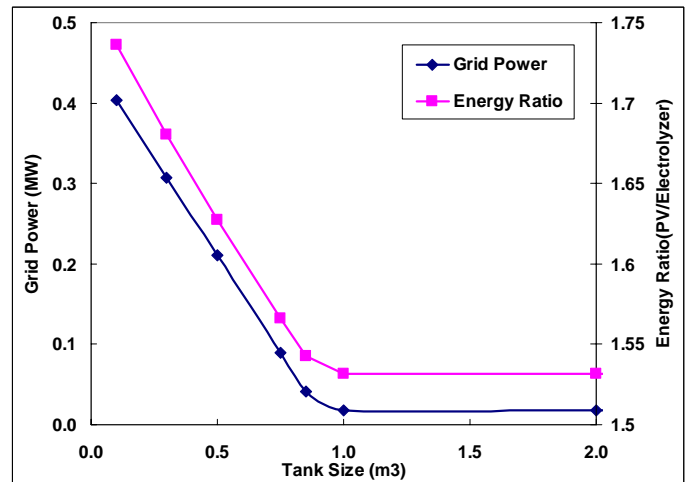


Fig. 11: Grid power and energy ratio (ratio of annual PV energy generated and annual electrolyzer energy consumed) as a function of tank size with PV and electrolyzer sizes held constant.

Figs. 12, 13, 14, 15 and 16 show the performance of the complete system for a PV array of 3.5 kW, electrolyzer size of 2.05 kW, a 1 kW fuel cell and a tank size of  $1 \text{ m}^3$  (33.5 kg). The summation of annual grid power for this system is 18 kW (positive denotes the system sent to the grid). Note that the annual balance for grid power is almost zero (0.4 % of annual load) and the hydrogen generated (104.5 kg) was completely consumed by the fuel cell.

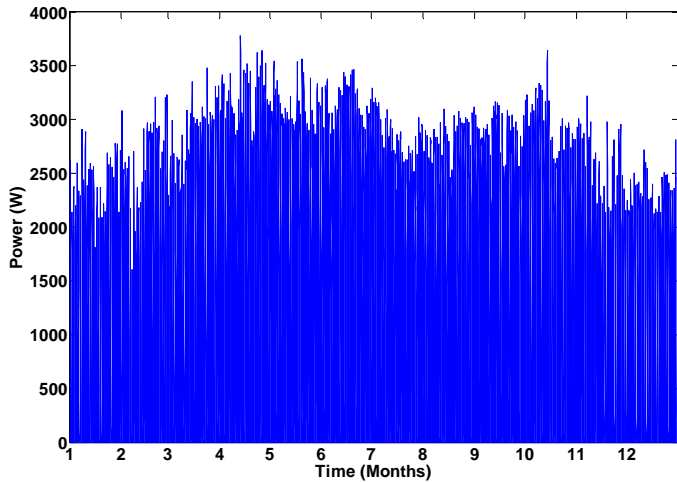


Fig. 12: Energy generated by PV over a period of one year for a tank size holding 33.5 kg of hydrogen when filled.

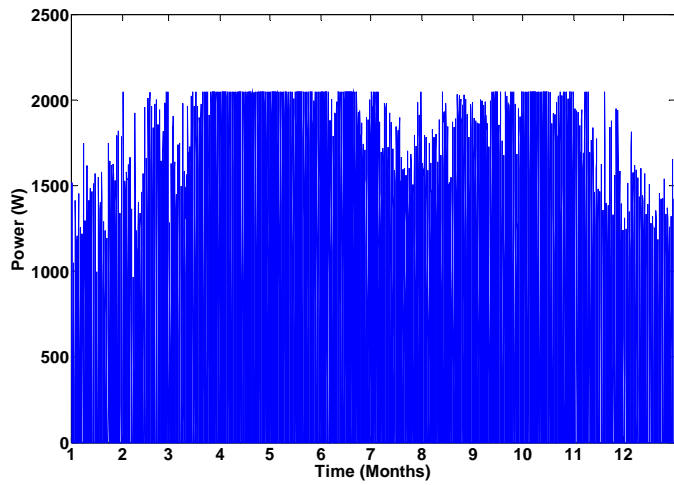


Fig. 13: Energy consumed by electrolyzer over a period of one year.

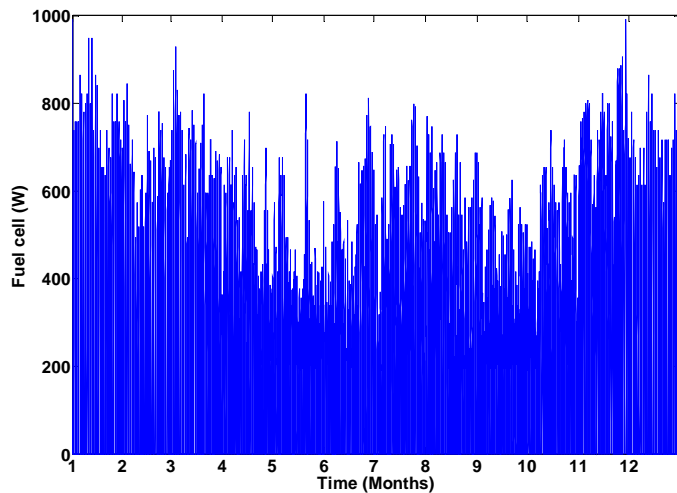


Fig. 14: Energy generated by fuel cell over a period of one year for a system with a tank size of 33.5 kg when filled.

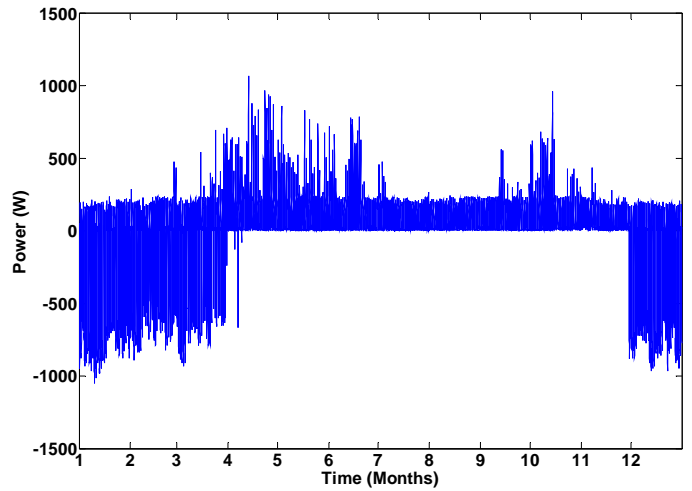


Fig. 15: Grid energy interaction over a period of one year for a system with a tank size of 33.5 kg of hydrogen when filled.

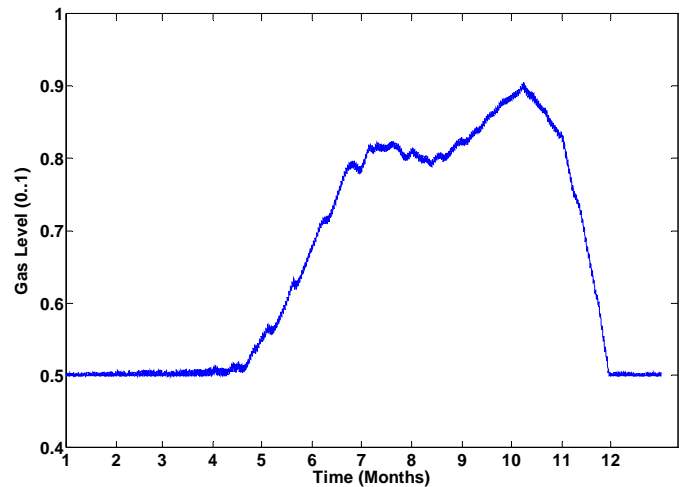


Fig. 16: Gas level in hydrogen storage tank (holding 33.5 kg of hydrogen when filled) over a period of one year.

## CONCLUSIONS

1. A system consisting of a PV array, an electrolyzer, a fuel cell and a compressor/tank has been modeled to examine its ability to furnish a house with a net zero amount of energy from the grid.
2. Power interchange with the grid was allowed in these calculations, but systems that resulted in net zero grid demand of the year were identified.
3. All calculations considered no excess hydrogen (beyond that used to supply the house with energy) to be generated over the year.
4. The results show that for smaller sized PV systems than optimal, capacity of the electrolyzer has little effect on the results, for the sizes considered here.
5. Excess hydrogen is generated in the summer months, filling the tank, and the level in the tank is drawn down in the winter.

## ACKNOWLEDGMENTS

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